

The authors thank the Science Research Council for its support of this work, including the construction of the Cambridge University 600 kV high-resolution electron microscope, as a joint project between the Cavendish Laboratory and the Department of Engineering, and for subsequent personal support. Laboratory facilities in Oxford were provided by Professor Sir Peter Hirsch FRS. The assistance of Mr E. Watanabe during the commissioning of the Oxford JEM 200CX is also appreciated.

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Acta Cryst. (1981), **A37**, 125–130

Secondary Processes Accompanying X-ray Diffraction. Thermal Diffuse Scattering

BY A. M. AFANAS'EV

I.V. Kurchatov Institute of Atomic Energy, Moscow, USSR

AND S. L. AZIZIAN

Yerevan State University, Yerevan, Armenian SSR, USSR

(Received 10 October 1978; accepted 18 May 1980)

Abstract

A general formulation of the theory of the secondary radiation yield by X-ray diffraction in a crystal is given which enables such processes as photoelectric effect, fluorescence and thermal diffuse scattering (TDS) to be described in a unified way. Expressions describing TDS by crystals are obtained with an account of their elastic properties. TDS specificity is analyzed in detail. It is

shown that the TDS yield curves are determined not only by the scattering cross sections for the incident and diffracted waves, but also by the phase relations between the scattering amplitudes.

1. Introduction

In the last 10–15 years, the study of the angular dependence of secondary radiation yields, *i.e.* for

photoelectrons, fluorescent radiation, the radiation due to thermal diffuse scattering, *etc.*, which arise during the interaction of X-rays with the atoms of a crystal under conditions of X-ray Bragg diffraction on the atomic planes, has met with ever increasing interest. The general theoretical analysis of the problem and a review of experimental work is given by Afanas'ev & Kohn (1978).

The angular dependence of secondary radiation yield is, first of all, strongly affected by the structure of the X-ray wave field in a crystal under dynamical scattering conditions. Besides, as has been recently shown, the yield curves of secondary photoelectrons (Kruglov & Shchemelev, 1976; Afanas'ev *et al.*, 1977) as well as those of fluorescent radiation (Andersen, Golovchenko & Mair, 1976) from foreign atoms introduced at a small depth into the crystal are extremely sensitive even to very small distortions of the layer adjacent to the surface of a crystal. This allows one to hope that the study of the secondary processes will continue to attract the attention of physicists engaged in the X-ray diffraction problem.

In the paper by Afanas'ev & Kohn (1978) the general theoretical approach to analysis of secondary processes accompanying X-ray diffraction, in both ideal and distorted crystals, was developed and the detailed analysis of the problem of photoelectron yield was carried out. The results of the developed theory can be directly used for the analysis of the fluorescent radiation yield. However, as far as TDS is concerned, more accurate analysis of the problem is required within the framework of the scheme developed by Afanas'ev & Kohn (1978). It is the treatment of this problem that served as an impetus toward writing this paper.

An attempt to develop a theory of the TDS yield was made by Annaka (1968). However, this paper presents an oversimplified treatment of the problem. Owing to the coherent character of diffraction scattering, the TDS cross section together with the terms proportional to the intensities of incident and reflected waves contains the interference term as well. The magnitude of the latter depends on the phase relations between the corresponding scattering amplitudes, and is not determined by the geometric mean as was adopted by Annaka (1968).

In § 2 the general statement of the problem of evaluating the secondary radiation yield is given which enables the photoelectric effect and fluorescent radiation to be analyzed as well as the TDS. Taking further application of the theory to TDS into account, we have confined ourselves to the case of an ideal crystal. In § 3, corrections to the coefficients of dynamical equations due to TDS processes are evaluated. In § 4, some typical situations of the angular dependence of the TDS yield are analyzed using concrete examples.

2. General theory of the yields of secondary processes on X-ray diffraction in ideal crystals

Let the X-rays be incident on a crystal at an angle close to the Bragg angle. Along with the incident wave, there also arises the diffracted wave in a crystal. We shall assume that the diffraction conditions hold true only for one of the reciprocal-lattice vectors (the so-called two-wave case of diffraction).

The wave field of X-ray radiation in a crystal can be represented in the form

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0(z) \exp(i\mathbf{k}_0 \cdot \mathbf{r}) + \mathbf{E}_h(z) \exp(i\mathbf{k}_h \cdot \mathbf{r}), \quad (1)$$

where \mathbf{k}_0 and \mathbf{k}_h are the wave vectors in the direction of the incident and diffracted waves, respectively. From the Maxwell equations one can readily obtain the following set of dynamical equations for the amplitudes $\mathbf{E}_0(z)$ and $\mathbf{E}_h(z)$

$$\begin{aligned} \frac{dE_0^m}{dz} &= \frac{ik}{\gamma_0} |\chi_{00}^{mn} E_0^n + \chi_{0h}^{mn} E_h^n| \\ \frac{dE_h^m}{dz} &= \frac{ik}{\gamma_h} |\chi_{h0}^{mn} E_0^n + (\chi_{hh}^{mn} - \alpha \delta^{mn}) E_h^n|. \end{aligned} \quad (2)$$

Here and later the indices that repeat themselves twice signify summation, $k = \omega/c$, the superscripts m, n mean the components of $\mathbf{E}_{0,h}(z)$, z is the depth of the reflecting layer within the crystal, $\gamma_{0,h}$ are the cosines of the angles between the vectors $\mathbf{k}_{0,h}$ and the inner normal to the entrance surface of the crystal. The parameter α defines the deviation from the Bragg conditions and $\chi_{hh'}^{mn}$ is the Fourier coefficient of the polarizability tensor which usually is represented in the form

$$\chi_{hh'} = \chi_{rhh'} + i\chi_{ihh'}.$$

Here $\chi_{rhh'}$ and $\chi_{ihh'}$ are determined by the real and imaginary parts of the X-ray scattering amplitudes, respectively. The $\chi_{ihh'}$ coefficients can be expressed as a sum of terms to each of which a definite process of X-ray absorption, *i.e.* photoelectric effect (Ph), thermal diffuse scattering and Compton scattering (CS), makes a contribution. In accordance with this,

$$\chi_{ihh'} = \chi_{ihh'}(\text{Ph}) + \chi_{ihh'}(\text{TDS}) + \chi_{ihh'}(\text{CS}). \quad (3)$$

The form of (2) differs, to some extent, from the standard form of dynamical theory equations. In fact, when analyzing diffraction scattering one usually assumes that

$$\chi_{00} = \chi_{hh}. \quad (4)$$

This approximation, which was used by Afanas'ev & Kohn (1978) for analyzing the secondary processes, is, as a rule, performed with good accuracy. In the general case, however, (4) does not hold for thermal and Compton scattering. It should be noted that the

contribution of these processes to the coefficients $\chi_{ihh'}$ is small and may be neglected when evaluating the wave fields in a crystal or the reflection and transmission coefficients. However, in the analysis of TDS and CS these corrections play the leading part and, therefore, the tensor form for the polarizability coefficients is preserved in (2).

To get the intensity of the secondary radiation yield let us consider a layer dz thick at depth z . The total absorption of X-rays in this layer is, evidently, determined by the difference between the incoming flux and outgoing flux, *i.e.*

$$\begin{aligned} \kappa(z) dz &= \gamma_0 |E_0(z)|^2 - |E_0(z + dz)|^2 + \gamma_h |E_h(z)|^2 \\ &\quad - |E_h(z + dz)|^2 \\ &= - \left[\gamma_0 \frac{d|E_0(z)|^2}{dz} + \gamma_h \frac{d|E_h(z)|^2}{dz} \right] dz. \end{aligned}$$

Now using (2), we easily obtain

$$\begin{aligned} \kappa(z) &- k \{ E_0^{m*}(z) \chi_{i00}^{mn} E_0^n(z) + E_h^{m*}(z) \chi_{ihh}^{mn} E_h^n(z) \\ &\quad + 2 \operatorname{Re} | E_0^{m*}(z) \chi_{i0h}^{mn} E_h^n(z) | \}. \end{aligned} \quad (5)$$

Making use of (3), we can separate out the contribution from each of the processes to the absorption. To do this one has, apparently, to replace χ_i in (5) by $\chi_i(A)$, *i.e.* by the quantity related to the process under consideration.

If the probability of the secondary radiation yield is defined by the function $P_A(z)$, then the intensity being recorded will be defined by

$$\kappa_A(\alpha) = \int_0^L dz P_A(z) \kappa_A(z, \alpha) \quad (6a)$$

in the case where the secondary radiation emerges from the entrance surface of a crystal; and

$$\kappa_A(\alpha) = \int_0^L dz P_A(L - z) \kappa_A(z, \alpha) \quad (6b)$$

when the secondary radiation leaves the crystal through the exit surface. Here L is the thickness of the crystal and $\kappa_A(z, \alpha)$ is determined by

$$\begin{aligned} \kappa_A(z, \alpha) &= k \{ E_0^{m*}(z, \alpha) \chi_{i00}^{mn}(A) E_0^n(z, \alpha) \\ &\quad + E_h^{m*}(z, \alpha) \chi_{ihh}^{mn}(A) E_h^n(z, \alpha) \\ &\quad + 2 \operatorname{Re} | E_0^{m*}(z, \alpha) \chi_{i0h}^{mn}(A) E_h^n(z, \alpha) | \} \end{aligned} \quad (7)$$

in accordance with (5). Here, in terms of the parameter α we introduce explicitly the dependence upon the angle of incidence of X-rays on a crystal that also figured in (2) and (5).

The problem of angular dependence of the secondary radiation yield is fully solved by formula (7). The specificity of some or other process is expressed through coefficients $\chi_{ihh'}^{mn}(A)$ and function $P_A(z)$. The

amplitudes $E_{0,h}$ do not, naturally, depend on the type of the recorded radiation and are defined only by the character of diffraction scattering. The analytical expressions for the field $E_{0,h}(z)$ can be found, for example, in the book by Pinsker (1978). As for photoelectric absorption, it should be noted that it makes the main contribution to the coefficients $\chi_{ihh'}$ whose structure is well known. These coefficients determined the fluorescent radiation yield (*cf.* Afanas'ev & Kohn, 1978). For the TDS process the corresponding contributions to $\chi_{ihh'}$ have not been calculated; this will be done in the next section.

3. Structure of $\chi_{ihh'}^{mn}$ (TDS)

The scheme for treatment of the angular dependence on the TDS yield is reduced to the following (*cf.* Annaka, 1968 and references therein). The incident direction of the X-rays is chosen (Fig. 1) so that for one of the reciprocal-lattice vectors (\mathbf{K}_{0h}) the Bragg condition is satisfied, while for the second one ($\mathbf{K}_{0h''}$), although it is not valid, the difference $|\mathbf{k}_0 + \mathbf{K}_{0h''}| - |\mathbf{k}_0| = q_{\min}$ is rather small, so that

$$q_{\min} \ll K_0. \quad (8)$$

Under these conditions purely elastic diffraction scattering does not, practically, occur; however, a strong one-phonon inelastic scattering in the direction of $\mathbf{k}_{h''}$ takes place. This process as well as any other process of absorption or inelastic scattering contributes to the coefficients of the dynamical theory.

The general scheme for the evaluation of corrections $\chi_{ihh'}^{mn}$ due to TDS was developed earlier by Afanas'ev & Kagan (1968) in connection with the problem of temperature dependence for the anomalous transmission effect. In this paper, the TDS occurring mainly in the vicinity of the directions of \mathbf{k}_0 and \mathbf{k}_h was analyzed. An account of the TDS process when there arises the third direction for intensive thermal scattering, as in the case

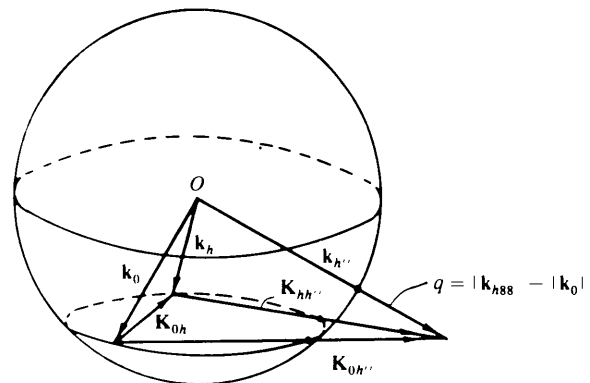


Fig. 1. The wave vectors \mathbf{k}_0 , \mathbf{k}_h , $\mathbf{k}_{h''}$ and the reciprocal-lattice vectors \mathbf{K}_{0h} , $\mathbf{K}_{0h''}$, $\mathbf{K}_{hh''}$.

under consideration, is taken within the framework of the scheme developed by Afanas'ev & Kagan (1968), although it requires very laborious intermediate calculations. Below, we give only the final result

$$\chi_{ihh'}^{mn}(\text{TDS}) = \frac{6\pi r_0^2}{\Omega_0 k} F(\mathbf{k}_h - \mathbf{k}_{h'}) F(\mathbf{k}_{h''} - \mathbf{k}_{h'}) \times \frac{T}{MC_s^2(h, h')} \ln \frac{q_0}{q_{\min}} \left(\delta^{mn} - \frac{k_{h''}^m k_{h'}^n}{k^2} \right), \quad (9)$$

where

$$\frac{1}{C_s^2(h, h')} = \frac{(\mathbf{k}_h - \mathbf{k}_{h'}) \cdot (\mathbf{k}_{h''} - \mathbf{k}_{h'})^q}{k^2} \times \frac{1}{3} \sum_j \int_0^{2\pi} \frac{V^l(j, \varphi) V^{q*}(j, \varphi) d\varphi}{C^2(j, \varphi)} \frac{d\varphi}{2\pi}. \quad (10)$$

In formula (9), $F(\mathbf{k})$ is the structure factor, M and Ω_0 are the mass and volume of the unit cell, respectively, T is the temperature in energy units, r_0 is the classical radius of the electron, q_0 has the value of the order of the limiting momentum of phonons. In formula (10), j labels three different acoustic modes, $C(j, \varphi)$ is the sound velocity, the integration being extended over all directions perpendicular to the vector $\mathbf{k}_{h''}$, V^l and V^q are the l and q components of the polarization vectors. Expressions (9), (10) allow complete analysis of the TDS specifically under diffraction conditions. As is seen from (9), (10), condition (4) is, apparently, violated here and the character of interference is determined by both the geometry of scattering (*i.e.* by mutual orientation of \mathbf{k}_0 , \mathbf{k}_h and $\mathbf{k}_{h''}$) and the anisotropy of the elastic properties of the crystal. The evaluation of the tensor $\chi_{ihh'}^{mn}(\text{TDS})$ is, in fact, reduced to calculation of the coefficients $C_s^{-2}(h, h')$ for which the dependence of the sound velocities and the polarization vectors in the direction of propagation of the sound wave in a crystal is assumed to be known. A more detailed analysis of this problem will be given in the next section. Here we shall only discuss the behaviour of the function $P_{\text{TDS}}(z)$ which determines the probability of the scattered X-rays emerging from a crystal. The behaviour of this function is given by a simple exponential law

$$P_{\text{TDS}}(z) = \exp \{-\mu_0 z / \gamma''\}, \quad (11)$$

where μ_0 is the linear absorption coefficient for X-rays of a given wavelength, $\gamma'' = \cos(\mathbf{k}_{h''}, \mathbf{n})$, \mathbf{n} is the unit vector normal to the entrance surface of a crystal. It should be noted that the scattered X-ray quanta may occur in the direction for which the Bragg condition is satisfied. In this case the probability of such quanta emerging from a crystal is, naturally, determined by another law and not by formula (11). However, the

fraction of such quanta for ordinary experimental conditions is negligibly small. Formulae (6), (7), (9) and (11) fully determine all the features of TDS. If one neglects in formula (10) the dependence of sound velocity on the direction in a crystal as well as the difference between the velocities of longitudinal and transverse waves [this is the approximation used in Annaka's (1968) paper], then the following simple expression for $\chi_{ihh'}^{mn}$ can be obtained

$$\chi_{ihh'}^{mn}(\text{TDS}) = A \left(\delta^{mn} - \frac{k_{h''}^m k_{h'}^n}{k^2} \right) g_{hh'}. \quad (12)$$

where

$$g_{hh'} = F(\mathbf{k}_h - \mathbf{k}_{h'}) F(\mathbf{k}_{h''} - \mathbf{k}_{h'}) \frac{T}{MC_s^2} \times \frac{(\mathbf{k}_h - \mathbf{k}_{h'}) \cdot (\mathbf{k}_{h''} - \mathbf{k}_{h'})}{k^2} \times A = \frac{6\pi r_0^2}{\Omega_0 k} \ln \left(\frac{q_0}{q_{\min}} \right). \quad (13)$$

In the paper by Annaka (1968), the approximation was made that

$$g_{0h} = g_{h0} = \sqrt{g_{00} g_{hh}}. \quad (14)$$

As is seen from formula (13), however, relation (14) is not valid. But in some particular cases considered in the paper mentioned above, the vectors $\mathbf{k}_h - \mathbf{k}_{h''}$ and $\mathbf{k}_{h'} - \mathbf{k}_{h''}$ are nearly parallel to each other; as a result, relation (14) holds with good accuracy and the error in the corresponding theoretical curves is not great. It is clear that various situations may arise when relation (14) is violated. Examples of this kind will be given in the next section.

One can easily take into account the difference between the velocities of the longitudinal ($C_{||}$) and transverse (C_{\perp}) waves, if the sound velocities themselves do not depend on the direction of propagation (the so-called case of an isotropic crystal). Under these conditions $g_{hh'}$ takes on the following form:

$$g_{hh'} = F(\mathbf{k}_h - \mathbf{k}_{h'}) F(\mathbf{k}_{h''} - \mathbf{k}_{h'}) \frac{T}{2M} \left\{ \left(\frac{1}{C_{||}^2} + \frac{1}{C_{\perp}^2} \right) \times \frac{(\mathbf{k}_h - \mathbf{k}_{h''}) \cdot (\mathbf{k}_{h''} - \mathbf{k}_{h'})}{k^2} + \left(\frac{1}{C_{||}^2} - \frac{1}{C_{\perp}^2} \right) \times \frac{|(\mathbf{k}_h - \mathbf{k}_{h''}) \cdot \mathbf{k}_{h''}| |(\mathbf{k}_{h''} - \mathbf{k}_{h'}) \cdot \mathbf{k}_{h''}|}{k^4} \right\}. \quad (15)$$

The presence of anisotropy does not allow an analytical expression for $g_{hh'}$, even for cubic crystals. If the anisotropy parameter is small, then the approximate expressions for the $g_{hh'}$ coefficients in a cubic

crystal may be written as

$$g_{hh'} = F(\mathbf{k}_h - \mathbf{k}_{h''}) F(\mathbf{k}_{h''} - \mathbf{k}_{h'}) \frac{T}{2M} \left\{ \left(\frac{1}{C_{44}} + \frac{1}{C_{11}} \right) \times \frac{(\mathbf{k}_h - \mathbf{k}_{h''})(\mathbf{k}_{h'} - \mathbf{k}_{h''})}{k^2} + \frac{C_{12} + C_{44}}{C_{11} C_{44}} \frac{|(\mathbf{k}_h - \mathbf{k}_{h''}) \mathbf{k}_{h''}||(\mathbf{k}_{h'} - \mathbf{k}_{h''}) \mathbf{k}_{h''}|}{k^4} + \frac{C_{11} - 2C_{44} - C_{12}}{C_{11} C_{44}} \times \left[\frac{(k_h - k_{h''})^x (k_{h'} - k_{h''})^x k_{h''}^x}{k^4} + \frac{(k_h - k_{h''})^y (k_{h'} - k_{h''})^y k_{h''}^y}{k^4} + \frac{(k_h - k_{h''})^z (k_{h'} - k_{h''})^z k_{h''}^z}{k^4} \right] \right\}. \quad (16)$$

Here C_{11} , C_{12} and C_{44} are the elastic constants of a cubic crystal.

The approximation formula (16) may be used to compute the coefficients for such crystals as Si and Ge.

4. TDS under Bragg diffraction conditions

Let us consider the case where diffraction in the Bragg geometry takes place, and the quanta scattered by the lattice vibrations emerge from the exit surface of a crystal. We shall consider the incident radiation on a crystal as being polarized and the crystal sufficiently thick that $L\mu_0 \gg 1$. The field distribution in the crystal is determined by the expressions

$$E_{0,h}(z, \alpha) = E_{0,h}(\alpha) \exp\left\{ \frac{ik \varepsilon(\alpha) z}{\gamma_0} \right\}, \quad (17)$$

$$E_0(\alpha) = E_{in}, \quad E_h(\alpha) = \sqrt{\beta} R_0(\alpha) E_{in},$$

where E_{in} is the incident wave amplitude,

$$\varepsilon(\alpha) = \frac{1}{2} |\chi_0 + \sqrt{\beta} \sqrt{\chi_h \chi_h} (y \pm \sqrt{y^2 - 1})|, \quad (18)$$

$$R_0(\alpha) = \frac{\sqrt{\chi_h}}{\sqrt{\chi_h}} (y \pm \sqrt{y^2 - 1}), \quad (19)$$

$$y = \frac{\alpha\beta - \chi_0(1 + \beta)}{2\sqrt{\beta} \sqrt{\chi_h \chi_h}}, \quad \beta = \frac{\gamma}{|\gamma_h|}. \quad (20)$$

The sign in front of the root in (18), (19) is determined by the condition $\text{Im } \varepsilon(\alpha) > 0$. If we substitute (17) and (11) in formulae (6a) and (7), then we obtain

$$\kappa_{TDS}(\alpha) = \frac{g_{00} + \beta P_R(\alpha) g_{hh} + 2\sqrt{\beta} g_{0h} \text{Re}[R_0(\alpha)]}{\mu(\alpha) + \mu_0/\gamma''} \quad (21)$$

with an accuracy of the nonessential constant factor. In this equation

$$\mu(\alpha) = \frac{2k}{\gamma_0} \text{Im } \varepsilon(\alpha), \quad P_R(\alpha) = |R_0(\alpha)|^2.$$

We recall that in the case of fluorescent radiation emerging from the atoms of the host crystal we have a formula analogous to (21) in which one should assume that

$$g_{00} = g_{hh} = 1, \quad g_{0h} = \varepsilon_h = \chi_{ih}/\chi_{i0}, \quad (22)$$

where χ_{ih} , χ_{i0} are the imaginary parts of the dynamical equations coefficients. As a result, the following expression can be obtained

$$\kappa_{FL}(\alpha) = \frac{1 + \beta P_R(\alpha) + 2\sqrt{\beta} \varepsilon_h \text{Re}[R(\alpha)]}{\mu(\alpha) + \mu_0/\gamma''}. \quad (23)$$

In accordance with formulae (18)–(20) there exists a correlation between the quantities $\text{Re}[R_0(\alpha)]$, $\mu(\alpha)$ and $P_R(\alpha)$, and each of them may be expressed in terms of the other two. Eliminating $\text{Re}[R_0(\alpha)]$ from (23), we easily obtain (cf. Batterman, 1964)

$$\kappa_{FL} = \left\{ |1 - P_R(\alpha)| - |1 - P_R(\alpha)| \frac{\mu_0/\gamma''}{\mu(\alpha) + \mu_0/\gamma''} \right\}. \quad (24)$$

The fluorescent curve asymmetry is mainly determined by the second term in (24) because of the presence of the factor

$$\frac{\mu_0/\gamma''}{\mu(\alpha) + \mu_0/\gamma''}.$$

[As a matter of fact, there is a weak asymmetry in $P_R(\alpha)$ as well, however, it does not practically affect the behaviour of the $\kappa_{FL}(\alpha)$]. In Fig. 2 the representative

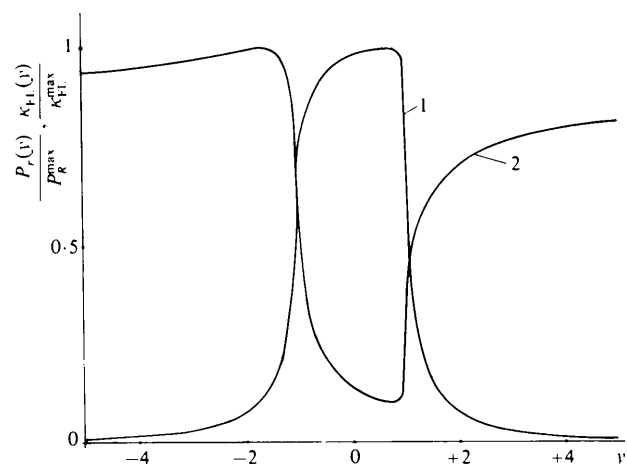


Fig. 2. Angular dependence of X-ray reflection coefficient $P_R(y)/P_R^{\max}$ (curve 1) and of fluorescent radiation yield $\kappa_{FL}(y)/\kappa_{FL}^{\max}$ (curve 2) for an ideal Si crystal on (111) reflection of Cu $K\alpha$ radiation.

curve of the fluorescent radiation yield is shown. The characteristic feature of such curves is that the intensity of the radiation yield for the large-angle side is a little higher than that for the small-angle side.

In fact, the character of asymmetry can be changed by detecting the radiation coming from the impurity atoms, but not from the host atoms (*cf.* Batterman, 1969).

In the case of TDS there is a greater variety of different behaviours of $\kappa_{\text{TDS}}(\alpha)$. This is because the relationships between g_{00} , g_{hh} and g_{0h} coefficients, as is seen from formulae (13)–(16), may vary within very wide limits. In cases where $g_{00} \approx g_{hh} \approx g_{0h}$ we have a situation close to the fluorescent one, *i.e.* the right-hand side intensity is higher than the left-hand side. One can also readily realize the situation when

$$g_{00} \gg g_{hh}^* \quad (25)$$

In this case

$$\kappa_{\text{TDS}}(\alpha) = \frac{g_{00}}{\mu(\alpha) + \mu_0/\gamma''}. \quad (26)$$

Comparing (26) with (24), one can easily see that the asymmetry is inversely changed. Asymmetry of this

* Physically, condition (25) means that the scattering cross section for the incident wave is much greater than that for the diffracted wave.

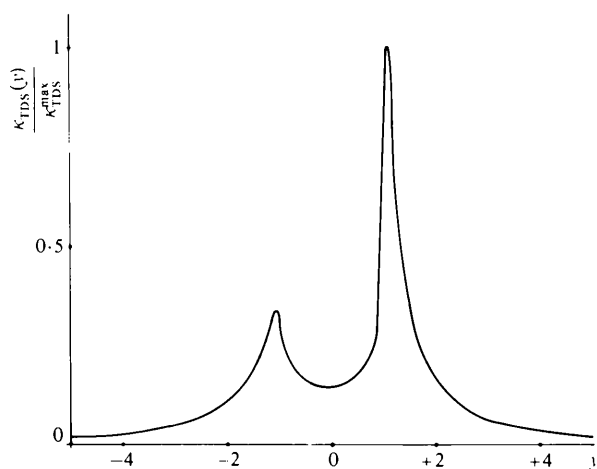


Fig. 3. Angular dependence of TDS yield $\kappa_{\text{TDS}}(\alpha)/\kappa_{\text{TDS}}^{\text{max}}$ under conditions $g_{hh} \gg g_{00}$.

kind has been observed and explained by Annaka (1968).

It should be emphasized that the inverse asymmetry can be observed when $g_{00} = g_{hh}$ and the g_{0h} coefficient has the negative sign. Such a situation is realized, for example, in the case where \mathbf{K}_{0h} and $\mathbf{K}_{0h''}$ correspond to planes (242) and (220), respectively. It is just this possibility that has not been taken into account in the above-mentioned paper, although it can be easily realized.

An interesting situation arises in the case where the condition opposite to (25) takes place. Here

$$\kappa_{\text{TDS}} \approx \frac{g_{hh} P_R(\alpha)}{\mu(\alpha) + \mu_0/\gamma''}$$

and, as is seen from Fig. 3, in the case under consideration the curve has two pronounced peaks, the right-hand peak being several times as high as the left-hand peak.

In reality, various intermediate situations may arise along with the limiting cases under discussion.

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